Phase Transitions: A New Paradigm for Evaluating Complexity in Learning and other Complex Systems

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The study of the computational complexity of algorithms takes a central place in computer science. Since its beginning, scientists have studied problems from the complexity point of view, categorizing their behaviour into a well-known complexity class hierarchy.

Many interesting problems have proved to be intractable. Among these, a class of problems that are particularly prone to a dramatic increase in computational complexity with increasing problem size is the class of combinatorial problems, which include learning problems.

Recently, it has been uncovered that computational problems show critical phenomena, similar to those that emerge in physical many-body systems. In particular, learning problems have shown to be the subject of the emergence of phase transitions, which allow a new paradigm for evaluating complexity to be considered. Thus, new paradigm is based on the notion of "typical" complexity instead of "worst case" complexity.

For computational systems, the discovery of a phase transition has important consequences. First of all, the phase transition region contains the most difficult problem instances, for which the computational complexity increases exponentially when the problem size increases. Then, the phase transition can be used as a source of "difficult" test problems for assessing the properties and the power of algorithms, and to compare them on meaningful problem instances on a parity base. Moreover, very small variations of the problem parameters may induce very large variations in the algorithm's behavior and/or in the types of solution. Then, the knowledge of the critical value allows the user to roughly predict the behavior. Moreover, by exploiting further the analogy with physical systems, it is possible to enter into the deep structure of the problem and of its solutions; the system's behavior near the phase transition allows a microscopic view of the solution space. This fact not only offers the possibility of a deeper understanding of algorithms' properties, but opens the way to the introduction of new effective algorithms.

In this talk, we will show the far-reaching effects of the presence of phase transitions in various learning approaches. Moreover, connections will be established with fundamental problem classes in computer science, such as the satisfiability problem and the constraint satisfaction problem classes, and with statistical physics approaches, which offered some very effective algorithms.

Finally we will show how phase transitions are rather ubiquitous in both natural and artificial systems, including human vision and neural system.

The subject is the content of a recently published book:
L. Saitta, A. Giordana, and A. Cornuéjols : *Phase Transitions in Machine Learning*. Cambridge University Press (Cambridge, UK, 2011)
(http://www.cambridge.org/gb/knowledge/isbn/item6049630/?site_locale=en_GB)